

Magic with cards

Introduction

This game can be thought of as a ‘magic show’, which can be performed by two people, who know about the secret of the game. They can ask anybody to pick up five cards from a pack of 52, and then one of them selects four cards and gives them to the other ‘magician’. The latter can then immediately tell what the fifth card is.

Explanation

While this magic show seems unbelievable, the mathematics it involves is rather simple. For convenience, we shall use the numbers 1 to 13 to denote the cards A, 2, 3, ..., 10, J, Q, K.

How the first magician chooses four cards out of the five

- Step 1: First of all, among the five cards picked up, at least two must belong to the same suit, according to the pigeon-hole principle. Take out these two cards. (If there are more than one possibilities, say, there are three cards of the same suit, take out any two of the same suit will do.)
- Step 2: Let the numbers on the two cards be x and y , with $x < y$. Let $y - x = d$. If $d > 6$, then hide the smaller card. If otherwise, hide the larger card. Note that to ‘hide’ a card means that the card will not be given to the partner. The remaining four will be given.
- Step 3: One of the two cards in Step 2 was not hidden. Put this card in the first position. (Note that the position of the four cards to be given to the partner is very important.)
- Step 4: Arrange the remaining three cards in order of ‘magnitude’. (As a convention, the spade is taken to be the largest, followed by the heart, the club and finally the diamond.) Let P be the smallest of the three cards, Q the one in the middle, R the largest one.
- Step 5: According to the value of d , arrange P , Q , R in order (as the second, third and fourth card). If $d > 6$, consider the value of $13 - d$ instead.

$$d = 1 \quad P, Q, R$$

$$d = 2 \quad P, R, Q$$

$$d = 3 \quad Q, P, R$$

$$d = 4 \quad Q, R, P$$

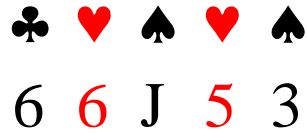
$$d = 5 \quad R, P, Q$$

$$d = 6 \quad R, Q, P$$

Example

We shall use the notations in the above explanations.

Suppose that the five cards chosen are



- Step 1: We have to take out two cards of the same suit, either two hearts or two spades in this case. As indicated above, which pair to choose does not matter. We will choose the pair of spades.
- Step 2: The ‘magnitudes’ of the two cards are 11 and 3, i.e. $x = 3$ and $y = 11$. Now $d = y - x = 11 - 3 = 8 > 6$, so we will hide the smaller card, i.e. spade 3.
- Step 3: The card not hidden in Step 2, i.e. spade J, is put in the first position.
- Step 4: Arrange the remaining three cards in order. From smallest to largest, they are heart 5, club 6 and heart 6. Hence P is heart 5, Q is club 6 and R is heart 6.
- Step 5: Now $d = 8$, so we consider the value of $13 - d = 13 - 8 = 5$. Thus we adopt the order R, P, Q , i.e. heart 6 is put in the second position, heart 5 the third and club 6 the fourth.

As a result, the following cards (in order) will be passed to the second magician:



How the second magician deduces the fifth (hidden) card:

- Step 1: The suit of the first card is the suit of the hidden card.
- Step 2: Consider the second, third and fourth cards, and identify P, Q, R according to the above notations. Hence the value of d can also be determined.
- Step 3: Add d to the magnitude of the first card. Let S denote this sum. S gives the magnitude of the hidden card. If $S > 13$, consider $S - 13$ instead.

Example

We use the above example. The four cards passed to the second magician are:



Step 1: Since the first card is a spade, the fifth (hidden) card must also be a spade.

Step 2: Among the second, third and fourth cards, heart 6 is the largest, so it is R . Heart 5 is the smallest, so it is P . Thus club 6 is Q . The order is R, P, Q . Thus d is 5.

Step 3: Add 5 to the magnitude of the first card to get S . Thus $S = 5 + 11 = 16$. But now $S > 13$, so we need to consider $S - 13$ instead. Now $S - 13 = 3$, so the magnitude of the hidden card is 3.

It can thus be deduced that the hidden card is spade 3.

Practice

1. Suppose you are the first magician. In each of the following sets of five cards, determine which four cards you should pass to your partner (the second magician) and in what order.

(a) ♠ ♥ ♥ ♣ ♦
4 3 7 9 K

(b) ♠ ♥ ♠ ♣ ♦
2 3 J 8 10

(c) ♠ ♥ ♥ ♣ ♣
8 8 6 A 8


(d) ♠ ♥ ♣ ♦ ♦
A A A A 2

2. Suppose you are the second magician. In each of the following sets of four cards passed from the first magician, deduce what the hidden card is.

(a) ♥ ♥ ♣ ♣
5 9 7 10


(b) ♠ ♣ ♦ ♥
J 8 10 3

(c) 
Q 7 5 6


(d) 
8 8 8 8

Answers

1. (a) 
3 9 K 4

(b) 
J 8 10 3

(c)  OR 
6 A 8 8 8 8 8 6

(d) 
A A A A

2. (a) Heart 8
(b) Spade 10
(c) Club 3
(d) Diamond A