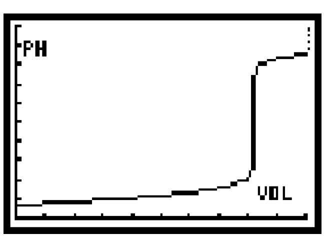
Modeling Titration Curves

A Point of Integration with Chemistry and

By Kent J. Crippen, Robert Mann, and David W. Brooks, Lincoln, Nebraska

Titration is a procedure used by chemists to determine the concentrations of dissolved chemical species. Chemistry teachers often use titrations to teach about acid/base behavior. The chemistry teacher uses the shapes of titration curves when explaining a variety of chemical behaviors. What a chemistry teacher describes as the buffering zone and the equivalence point of a titration curve, the mathematics teacher describes as inflection points of a function. The titration curve represents a possible way to connect these two disciplines.

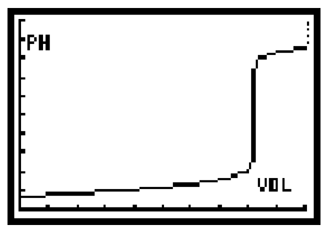
We have developed a simple calculator program that generates a titration curve for any reacting acid species with a strong base (such as sodium hydroxide). The user enters these quantities: molarity of acid; dissociation constant for the acid (Ka); dissociation constant for water (Kw); volume of acid to be titrated; and molarity of titrant (such as NaOH). The calculator then generates a titration curve plotting pH vs. volume titrant added (Figure 1).



Kw = 1.0x10^-14
Ma = 0.1 M
Mb = 0.1 M
Va = 100 mL
Ka = 100

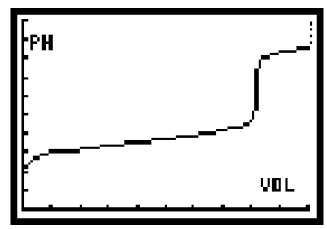
Figure 1

The program allows students to generate multiple titration curves without performing the actual titration. The chemistry instructor can then focus instruction on the differences between curves as seen in Figures 2-4. These are interpreted in terms of the chemical behavior of the reacting acid.



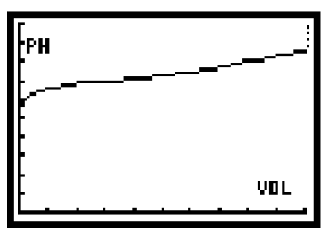
Ka = 100

Figure 2



Ka = 1 x 10-5

Figure 3

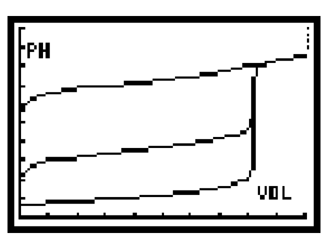


Ka = 1 x 10-10

Figure 4

Kw = 1.0x10^-14
Ma = 0.1 M
Mb = 0.1 M
Va = 100 mL

Using the Titrate program*, the math teacher can generate multiple samples for analysis of various slopes and critical values (Figure 5).



Ka1 = 1 x 10^-10
Ka2 = 1 x 10^-5
Ka3 = 100

Kw = 1.0x10^-14
Ma = 0.1 M
Mb = 0.1 M
Va = 100 mL

Figure 5

In pre-calculus, an exciting extension using this program would be a "find the function" activity for students. Since the titration curves are generated by a program using data manipulation and not an equation, the students are given a graph, but no function. A terrific problem-solving opportunity has emerged. Instruct the students to find a function which will generate a curve which best matches the titration model provided. Have them then use this curve to predict pH values for unknown volumes and volumes for currently unknown pH values. They can then check their predictions by slightly altering the range and/or scale of the original program. As the students try to create a function with a matching graph, they should gain valuable knowledge about function properties, transformations, and piece-wise functions. They should also learn how to better manipulate their calculators and how to interpret function inputs and outputs. As the students explore this open-ended question, you may give as little or as much guidance as you like. You may, for instance, suggest that the students try piece-wise and radical functions. The following function modeled one titration curve very well (Figure 6):

y= 5(x-8)^(1/7) + 8

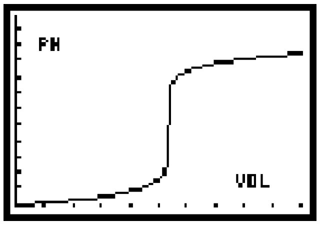


Figure 6

Follow-up discussions should include basic function concepts as well as the effects of stretches and shifts upon the original curve.

Another extension would require the students to adapt the program (or the current lists) to find the derivative of these titration curves. This would allow the students to talk about the inflection points of the original curves and how they relate back to the scientific experiment that developed them. The point of maximum slope is of particular importance since this is the equivalence point—the point where the reactants are equal in quantity. By analyzing a variety of curves and determining the point of maximum slope (the derivative of the function), students discover that equivalence points are not the same for all titrations. More importantly, that chemical behavior, specifically the ionic character (or degree of dissociation into ions) of the reacting specie dictates the point of equivalence.

This program is an excellent tool for teachers integrating Chemistry and Mathematics. Students can use the program to investigate the functions and critical values inherent to a titration curve, or to explain those functions and values based upon the nature of reacting compounds. In either case, the mathematics and chemistry are obviously intertwined. Instruction that emphasizes both the Chemistry and Mathematics will help the student develop a more thorough understanding of this phenomenon.◆

Kent J. Crippen, *kcrippen@lps.esu18.k12.ne.us, Lincoln Southeast High School, Lincoln, Nebraska*
Robert Mann, *rmann@lps.esu18.k12.ne.us, Lincoln Northeast High School, Lincoln, Nebraska*
David W. Brooks, *dbrooks@unlinfo.unl.edu, University of Nebraska-Lincoln, Lincoln Nebraska*

*The 82/83 program can be downloaded from
ftp://archive.ppp.ti.com/pub/graph-ti/calc-apps/
http:///www.ti.com/calc/docs/80xthing.htm,
or **http://www.cci.unl.edu/calculators/titrate.html.**

Correcting a Calculator "Lock-Up"

by Terry Walsh

Recently, my TI-92 locked up and I couldn't get it to operate, even after I put in brand new batteries. Contacting TI, I received the following suggestions to try:

1. Pull out one of the batteries.
2. Hold down the ON key for at least 10 seconds.
3. Replace the battery.
4. Turn the calculator on and adjust the contrast ("diamond" and "+" keys on the TI-92 or 2nd ↑ for the TI-8X series).
5. If these four steps don't work, on the TI-92, try the reset sequence of lock, diamond and on.

The first four steps can be tried on any of the TI graphing calculators. If your calculators ever freeze, try these steps if nothing else works. If it is still frozen, call 1-800-TI-CARES, because they do and they'll help you troubleshoot the difficulty.