

A Macroscopic Quantum Manifestation

Superconductivity was first observed in mercury cooled to $\sim 4.2^\circ\text{K}$ by Kamerlingh-Onnes in 1911. Now it is known that there are many other materials - such as metals & ceramics - that exhibit the property of superconductivity, such as Molybdenum ($T_c = 0.92^\circ\text{K}$), Vanadium ($T_c = 5.3^\circ\text{K}$) and Tungsten ($T_c = 0.012^\circ\text{K}$). Mercury was also the substance in which the isotope effect was observed. The isotope effect is a

relation, $T_c M^{\frac{1}{2}} \equiv \text{const.}$ where M is the ion mass in the lattice medium and T_c is the critical

temperature at which the material transits into the superconducting state. Indeed for many years after this curious observation many of the finest minds in physics grappled with this problem but in vain. They included Einstein, Bloch, Heisenberg and Landau, and only in 1957 did a microscopic theory of superconductivity emerge: The BCS theory for weak carrier-excitation coupling.

What exactly causes superconductivity? The answer may be rather surprising. According to the theory formulated by Bardeen, Cooper and Schrieffer when certain substances (metals are being focussed upon, unless otherwise stated) are cooled to below a certain temperature T_c thermal motions are sufficiently weak to permit the formation of electron pairs due to the distortion of the lattice medium. This can be visualized as follows: an electron moving in a sufficiently cooled lattice medium distorts the latter due to its charge, and there will be a “coat” of positive charge surrounding the electron. This permits another electron to be attracted to this negative-positive charge system. Moreover, since the ions in the lattice medium are “oscillating” (even at absolute zero) due to the uncertainty principle, any passing electron will excite these oscillations and correspondingly the ionic lattice. As the ions in the lattice medium fall to its initial ground state energy in the form of phonons are radiated this, will in turn strengthen the inter-electron attraction. This BCS theory however, only works well for substances with weak coupling: λ less than 1.5. What is λ ? λ is actually a coupling constant that is non-universal among the various superconductors, and λ is the coupling constant that determines the strength of the carrier-excitation interaction. This is a very important function, and its calculation for different substances is very complex and is related to the Eliashberg equation. Let us introduce a few basic properties of superconductivity:

1. The band theory of solids is extremely important in understanding superconductivity, although a few modifications may have to be made, according to the different properties of the superconductor. In this sense then the Fermi energy and the Fermi velocity are also very important. The Fermi velocity can be

calculated as $v_F = \sqrt{\frac{2E_F}{m}}$, with $E_F = -\alpha(T)kT$ with E_F as the Fermi energy. Knowing the

Fermi velocity enables us to calculate the coherence length, or the size of the electron pair as $\frac{\hbar v_F}{\pi\Delta}$. In

typical superconductors that are not heavy fermions, organic in nature and whose λ value is less than 1.5 the coherence length is approximately 10^{-4} cm, at inter-atomic lengths. Superconductivity depends quite a lot on the Fermi surface, in fact, most of the bound electrons in Cooper pairs “originate” from near or at the Fermi level, hence there is a high density of states here.

2. The critical temperature should be the most well-known term in superconductivity, and it is calculated by

$T_c \approx 1.14\hbar v_m e^{-\frac{1}{\lambda}}$ with v_m as the Debye frequency and calculated as

$v_m = \left(\frac{3N}{4\pi V}\right)^{\frac{1}{3}} v_s$ with $\frac{N}{V}$ as the atom density and with the speed of sound as v_s . In

isotropic models one can also use $\Delta_0 = 1.76T_c$.

3. Often a superconductor is characterized by the presence of energy gaps. This energy gap is calculated

as $\Delta_0 = 2\hbar v_m e^{-\frac{1}{\lambda}}$ with Δ_0 as the value of the energy gap at 0 K. This energy gap will be one of the most important topics for discussion.

4. Another important term to know would be the coherence length, or the length of a Cooper pair. This can vary in different classes of superconductors, and of course this is also a factor that can affect other

phenomena, such as radiation absorption. The coherence length is calculated by $\xi = \frac{\hbar v_F}{\pi \Delta}$.

Many interesting phenomena related to superconductivity exist. Let us have a look.

When a magnetic field is applied to a metal (sometimes magnetic impurities can cause similar effects), electronic states restructuring will occur, and so will the Fermi levels. In this sense there can arise superconductors with multi-gap, “overlapping” energy bands and “gap-less” structures. Gaps are energy intervals in which electrons can possess a certain discrete amount of energy. In superconductors with overlapping bands there are 2 groups of carriers with different properties, most notably different energy values. In this case, although the 2 groups of electrons can actually make inter-band transitions, inter-band Cooper pairing cannot actually occur. Although there are now 2 distinct groups of electrons there is no 2 different “parts” of the superconductor, so all the basic properties of the superconductor, such as the T_c of the superconductor. In this sense the 2 bands act upon each other. In “gap-less” superconductors, as the name suggests, there is no energy gap, and the electrons can possess any amount of energy. This can come about when any external agent tries to break up the bound Cooper pairs, such as an external magnetic field that is sufficiently strong, or magnetic impurities. When Cooper pairs dissociates, there will be single electrons who can occupy other states needed for the transmission of current, for example. This is an excellent example of the fact that an energy gap is not always needed for the transition to a superconducting state to occur. Moreover, in such a situation the electromagnetic radiation absorption of the superconductor is very dependent on the existence and the value of the energy gap, since electromagnetic radiation absorption only occurs at above a certain threshold

frequency $f = \frac{2\Delta_0}{\hbar}$ at 0 K. For electromagnetic radiation absorption in a superconductor at finite temperatures,

there are thermal excitations in the “normal” phase, in terms of the 2-fluid model, which will allow absorption at frequencies less than the Cooper pair break-up energy, which in any case will further promote electromagnetic radiation absorption since at above the threshold frequency the dissociation of Cooper pairs occurs and henceforth absorption is enhanced.

The Josephson direct-current and alternating-current effects are probably the most well-known of all phenomena in the field of superconductivity. The dc Josephson effect is that a superconducting dc can tunnel through a dielectric with no externally applied potential difference in a superconductor-dielectric-superconductor system with the 2 superconductor layers made of different superconductors. This effect arises when the phase of electrons are coherent within the same superconducting layer but different in the other superconducting layer., and obviously to attain equilibrium a current must flow, and in this case the current is known as the Josephson current.

The Josephson ac current is also interesting. The alternating current is generated when a constant voltage is applied and the interesting point to note is that this ac Josephson current will vanish when an integral number of magnetic

flux quantum (integral value of $\frac{hc}{2e}$) passes through such a S-I-S setup (so obviously the setup has to be placed in a

constant magnetic field) as the phase of the magnetic flux current exactly cancel out the current. The ac Josephson current has been experimentally observed; electromagnetic radiation will be emanated when there is an ac current and

in this case with a frequency of $f = \frac{2eU}{\hbar}$, with U being the externally applied voltage.

There thermal properties of superconductors are also interesting. The total net thermal conductivity is the sum of the thermal conductivities of the electronic system and the lattice system $\kappa_{total} = \kappa_{electronic} + \kappa_{lattice}$ *, and for

each component:
$$\frac{1}{\kappa_{\text{electronic}}} \approx \frac{1}{\kappa_{\text{electron-electron}}} + \frac{1}{\kappa_{\text{electron-lattice}}} + \frac{1}{\kappa_{\text{electron-impurity}}}$$

and

$$\frac{1}{\kappa_{\text{lattice}}} \approx \frac{1}{\kappa_{\text{lattice-lattice}}} + \frac{1}{\kappa_{\text{lattice-electron}}} + \frac{1}{\kappa_{\text{lattice-impurity}}}$$

The calculation for each κ value is complex. The lattice thermal conductivity is interesting when the material transits to the superconducting state; instead of showing a linear dependence on the temperature a peak is observed in the graph. This is due to the reduction of the number of chaotic motions of the entire electron-lattice system, and the effective increase in the phonon free path. In this sense, the thermal conductivity obviously increases since the phonons can more effectively conduct heat even though the total phonon density is decreasing. Soon, as the phonon mean free path increases to beyond a critical value related to electronic scattering and other forms of scattering, such as phonon-impurity and phonon-lattice begins to affect the phonon thermal conductivity. In effect, doping a superconductor may affect the thermal conductivity of a superconductor.

The electromagnetic effects of superconductors are interesting as well. For example, a surface current is induced by an externally applied magnetic field and interestingly, the relation between the current and the external magnetic field is spatially nonlocal. This is expected, of course, since the electrons in the Cooper pairs are spatially correlated over distances that are large, even by atomic standards - inter-atomic distances are $\sim 10^{-4}$ cm. Superconductors can be classified here, again, into 2 different groups: London and Pippard type, where the magnetic field penetration depth is large as compared to the coherence length and where the magnetic field penetration depth is small as compared to the coherence length and of course, the current-field relationship is nonlocal. According to the

Ginzburg-Landau theory superconductors are grouped into 2 classes: one with $\kappa < \sqrt{2}$, and the other with $\kappa > \frac{1}{\sqrt{2}}$.

In Type 1 superconductors they are the ones with low critical field values, and there is absolute magnetic field screening by the induced surface currents, and when the field manages to penetrate the superconducting sample superconductivity is destroyed. Type 2 superconductors however, allow the magnetic field to penetrate into the sample if it attains a certain lower critical field value (there is the corresponding upper critical field value where superconductivity is indeed destroyed), but the sample remains superconducting. Of course this implies the total expulsion of the external magnetic field when the magnetic field is below the lower critical field limit. Now when the magnetic field penetrates the superconducting sample electrons moving perpendicular to the magnetic field lines will begin to possess circular orbits around a central axis; these are the vortex lines. Inside these vortices the sample is in the normal state, and for such pure (or possessing point-like impurity centres) mixed-state superconductors the critical current is infinitesimal - negligible. This is due to the Lorentz force acting on the vortices and leading to energy dissipation and consequently losses. When such superconductors possess large impurity centres the critical current value increases as vortex pinning can occur on the impurity centres, and the Lorentz force required to rip the vortices off the impurity centres increases correspondingly. Such superconductors are termed "hard", or type 3, superconductors.

There is an effect that is known as the proximity effect. When there is a superconductor-normal conductor junction the Cooper pairs can actually tunnel into the normal conductor layer, such that the N film will exhibit weak superconductivity. Of course, the degree at which superconductivity is observed in the N film depends on the quality of junction, and thickness of each of the S and N layers. This is induced superconductivity.

In terms of the crystal structure of superconductors, they can again be split into 2 groups: A15 and the B1 group. A15 superconductors were the record holders in terms of their T_c , and their T_c is strongly dependent on their stoichiometry. A look at their crystal structures will tell us why.

As it can be seen, the A15 superconductor crystal structure is made up of chains and cubes. A represents the positions of the atoms of the transition elements, while B represents the positions of the atoms of semi-metals or poor metals. One can hence see why the T_c of such superconductors are so stoichiometric-dependent. If the stoichiometry

$A:B$ is not 3:1 disorder in the crystal structure will set in and the phonon absorption and scattering will be affected. B1 superconductors are basically more resistant than the A15 superconductors, and some of them have a high critical temperature, too, just like the A15 superconductors. Both of these types of superconductors, however, are very brittle.

Superconductors may not necessarily be metallic, some of them are even organic, such as the Bechgaard salts described by the formula $(TMTSF)_2 X$, with *TMTSF* as Tetramethyltetraselenafulvalene and *X* as an inorganic ion such as Chlorine (monovalent), and the $(BEDT - TTF)_2 X$ molecule, with the same *X* but with *BEDT-TTF* as bistetrathiafulvalene. In recent years there have been much interest (the flurry of excitement in this field - high-temperature superconductivity, with or without copper - was undeniably overwhelming) in the production of high-temperature superconductors. Before the discovery of the famous cuprates the primary progress in this entire field was made in niobium compounds, which is why when the cuprates' superconducting properties were announced by Bednorz and Müller scientists were surprised. The current record holder at 125 K is the Thallium-Barium-Calcium-Copper Oxides discovered by Hermann & Sheng. There are still many questions than answers currently, but much work has been made. In addition to high-temperature superconductivity, there are many other interesting fields of superconductivity in which much progress has also been made - for example, heavy fermion superconductivity. The current aim, of course, is to increase T_c , only with that increased can we turn all superconducting electronics into reality, enhancing digital electronics.