An Introduction to Black Holes

Stars of mass greater than ~4.5 sm., they will usually end their lives as a pulsar or a black hole. Before a star has exhausted its nuclear fuel, the inwards compression of gravity & the outwards expansion forces of nuclear fusion are in balance (the outward pressure of a star may be calculated by $P=nkT$, for n being the number density, k being the Boltzmann's constant, T being the temperature, and P being the pressure). When a star has exhausted its nuclear fuel gravity starts to dominate, compressing the core of the star and raising temperatures, resulting in the rapid production of neutron-rich elements.

A supernova occurs, which is caused by the shockwave emanating from the stable iron core (for giants & supergiants) due in turn to the collapsing outer shells of the star (Newton's 3rd Law). For such massive stars, they can end their lives, as mentioned above, as neutron stars. Neutron stars (and pulsars) are prevented from collapsing into a singularity by neutron degeneracy pressure, as opposed to the electron degeneracy pressure that supports white and black dwarves. Neutron degeneracy pressure can be calculated by:

$$p_{\deg} = \frac{\hbar^2 \pi^3}{15 m_n} \left(\frac{3n}{\pi}\right)^{\frac{5}{3}}$$

for n being the density of the neutron gas, m_n is the neutron rest mass, and with $\hbar = \frac{h}{2\pi}$. When a neutron star forms a large number of electron neutrinos are released; this is *not* beta decay though, neutrons are formed, they are not decaying. However, neutron degeneracy pressure, though much stronger than electron degeneracy pressure, cannot hold up the collapsed (or, collapsing) star if the star's mass is >7.5 to 8.0 stellar mass. A singularity forms, and an event horizon is created. Let us have a look at some black hole properties:

1. For a Kerr black hole, the horizon area can be calculated by:

$$A = \frac{8\pi G^2 M^2}{c^4}\left[1 + \sqrt{1 - \left(\frac{cJ}{GM^2}\right)^2}\,\right]$$

for J being the momentum, G being Newton's gravitational constant, M is the mass of the black hole.

2. The Schwarzschild radius of a Schwarzschild black hole can be calculated by:

$$R_s = \frac{2G}{c^2} M$$

for M being the mass of the black hole, G being Newton's gravitational constant.

3. As black holes have a temperature that evolves exponentially with age, to calculate the temperature of a Schwarzschild black hole:

$$T = \frac{6 \times 10^{-8})}{M}$$

with M being the mass of the black hole.

4. The approximate lifetime of a Schwarzschild black hole can be calculated as:

$$\tau_{life} \sim 10^{66} M^3 \text{ years}$$

with M being the mass of the black hole.

5. Of course, do not forget the mass of the black hole, being:

$$M = \frac{C_{orbit}}{2\pi G P_{orbit}{}^2}$$

with G being Newton's gravitational constant, C_{orbit}, P_{orbit} being the circumference and the period of orbit respectively. This implies that one would have to orbit the black hole to calculate the mass of the black hole.

6. Considering an ideal black hole, the density would be $\frac{3c^6}{32\pi G^3 M^2}$.

7. Black holes, like all other heavenly bodies, can form into binary pairs. Hence the orbital period can be calculated by:

$$P_{orbital} = 2\pi\sqrt{\frac{d^3}{2GM}}$$

with G being Newton's gravitational constant, M being the mass of the black hole, d being their separation.

8. After some time the orbiting black holes will merge, and this time t can be calculated by:

$$t = \left(\frac{5}{512}\right)\left(\frac{c^5}{G^3}\right)\left(\frac{d^4}{M^3}\right)$$

with G being Newton's gravitational constant, M being the mass of the black hole, d being the separation between the 2 black holes.

9. Since pair production is an inevitable consequence of the Uncertainty Principle it follows that when this occurs in the ergosphere of a Kerr black hole the momentum of the Kerr black hole decreases, as one of the particle enters the theoretical outer event horizon. In principle this is the extraction of energy from the Kerr black hole, which can be calculated by:

$$M_0c^2\left(1 - \frac{1}{\sqrt{2}}\right) \sim 30\%$$

Now that we have looked at some equations describing some basic properties of black holes, let us consider what happens when an object falls into the black hole. First of all, the object will be distended, and its length is $l = \sqrt{\frac{2M}{R}}(t_1 - t_2)$, for R being the Schwarzschild radius, M being the mass of the black hole, t_1, t_2 being the time coordinates with which the top & bottom ends of the object fall towards the singularity, respectively. The reason why there is a time difference is due to the highly warped spacetime in the vicinity of the black hole. The object will take $1.54 \times (10^{-5}\frac{M_h}{M_s})$ seconds to hit the singularity. M_h, M_s are the masses of the black hole and our sun respectively. Finally, the volume of the distended object will be $V = \frac{1}{R^{\frac{3}{2}}}$, with R being the radius of the distended object.

Theoretically, Kerr-Newman black holes have an ergosphere, an inner and outer event horizons, and an electric charge. For Kerr black holes, they have only an ergosphere, 1 event horizon. For a Reissner-Nordström black hole, it only has an electric charge and 2 event horizons. "In between" the 2 event horizons theoreticians believe that the spacetime is flat. Of course, in these 3 classes of black holes a singularity is present, but in Kerr and Kerr-Newman black holes the singularity may not be a point but rather a hoop. This is allow for any ordinary matter to pass through without experiencing immensely powerful gravitational tidal forces. Of course, this still has the element of speculation, but one may never know.